

# *Ab Initio* Calculations on Large Molecules Using Molecular Fragments. Benzene and Naphthalene Isomer Characterizations and Aromaticity Considerations<sup>1</sup>

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**Abstract:** An *ab initio* investigation of several benzene and naphthalene isomers and hexatriene has been carried out. It was found that the relative order of total energies for the benzene isomers is benzene > fulvene > 2,3-dimethylenecyclobutene > trimethylenecyclopropane > Dewar benzene. In the naphthalene isomer series, the relative order of total energies was found to be naphthalene > azulene > fulvalene. Examination of the electronic structure of the various isomers permits new insight to be gained into the concept of aromaticity, and also provides a rationalization for some of their chemical behavior in terms of their molecular orbital structure.

Throughout the past century the rationalization of the differences in stability and reactivity of aromatic molecules compared to their open-chain analogs has been recognized as one of considerable importance. Several suggestions as to the most appropriate definition to be used to describe the aromaticity of a given molecule have been proposed<sup>2-6</sup> which use a variety of different criteria for distinguishing the degree of aromaticity that is present. In each of these proposals, a detailed understanding of the electronic and geometric structure of the molecule in question is essential, either explicitly or implicitly, to the development of the proposal.

In the past, many semiempirical techniques have been employed to quantify these notions.<sup>7-9</sup> Despite their several limitations, they have provided considerable insight into the concepts of resonance, delocalization, and aromaticity. However, many of these limitations can be eliminated by the use of *ab initio* quantum mechanical techniques, in which all electrons and nuclei are considered explicitly.

In this study, an *ab initio* technique<sup>10,11</sup> has been applied to several isomers of benzene and naphthalene. 1,3,5-*trans*-Hexatriene, although not an isomer of benzene, has also been included in this study because of its importance to discussions of aromaticity. The *ab initio* method using molecular fragments that is employed in this study is particularly appropriate, for it

has been found to be one which allows the determination in a practical manner of the electronic structure of reasonably large molecules of arbitrary geometry. Furthermore, the results are readily interpretable into concepts familiar to chemists. Also, previous studies on benzene itself<sup>11</sup> and other hydrocarbons indicate that the molecular orbital energy level ordering, molecular geometry, Hellman-Feynmann forces and fields at nuclei, and other properties are well predicted.

The isomers chosen for this study do not exhaust all of the possible isomeric forms of benzene and naphthalene. Rather, the particular molecules chosen for inclusion were those in which a structure determination or extensive geometry search had been carried out, either on the parent hydrocarbon or a closely related substituted hydrocarbon. This procedure allows examination of the difference in electronic structure for the molecules as they actually exist, and eliminates perhaps misleading hypothetical structures from consideration.

## Theoretical Procedure

The detailed development of the procedure to be used has been given previously<sup>10,11</sup> and will only be summarized briefly here. The basis orbitals that are employed are normalized floating spherical Gaussian orbitals (FSGO), defined by

$$G_i(r) = (2/\pi\rho_i^2)^{1/4} \exp\{-[(r - R_i)/\rho_i]^2\} \quad (1)$$

where  $\rho_i$  is the orbital radius and  $R_i$  is the location of the FSGO, relative to some arbitrary origin. When  $\pi$ -type orbitals are used, a linear combination of two FSGO's is employed, *i.e.*

$$G_\pi = (G_u - G_d)/[2(1 - \Delta_{ud})]^{1/2} \quad (2)$$

where  $G_u$  and  $G_d$  are FSGO's that are symmetrically placed above and below the plane of atoms, on a line perpendicular to the central atom. The nonlinear parameters for these orbitals are variationally determined by studies on molecular fragments. The parameters for the two molecular fragments of particular interest to these studies,  $\text{CH}_4$  and  $\cdot\text{CH}_3$ , are given in Table I. These fragments were found to be particularly suitable for hydrocarbon investigations in a detailed earlier study<sup>11</sup> of various fragment possibilities.

(1) This work was supported in part by grants from the University of Kansas, the Upjohn Co., Kalamazoo, Mich., and the Deutscher Akademischer Austauschdienst.

(2) (a) See, for example, M. J. S. Dewar and C. DeLlano, *J. Amer. Chem. Soc.*, **91**, 789 (1969), and earlier papers in this series; see also M. J. S. Dewar, "The Molecular Orbital Theory of Organic Chemistry," McGraw-Hill, New York, N. Y., 1969; (b) A. Streitwieser, "Molecular Orbital Theory for Organic Chemists," Wiley, New York, N. Y., 1961.

(3) J. F. Labarre and F. Crasnier, *J. Chim. Phys. Physicochem. Biol.*, **64**, 1664 (1967).

(4) R. W. Hakala, *Int. J. Quantum Chem., Suppl.*, **1**, 187 (1967).

(5) A. Julg and P. Francois, *Theor. Chim. Acta*, **8**, 249 (1967).

(6) D. P. Craig, "Theoretical Organic Chemistry (Kekulé Symposium)," Butterworths, London, 1955, pp 20-34; see also *J. Chem. Soc.*, 3175 (1951).

(7) See, for example, P. B. Emedocles and J. W. Linnett, *Theor. Chim. Acta*, **4**, 377 (1966), and ref 8 and 9; see also ref 2 and 4.

(8) J. D. Roberts, A. Streitwieser, Jr., and C. M. Regan, *J. Amer. Chem. Soc.*, **74**, 4579 (1952).

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(10) R. E. Christoffersen and G. M. Maggiora, *Chem. Phys. Lett.*, **3**, 419 (1969).

(11) R. E. Christoffersen, D. W. Genson, and G. M. Maggiora, *J. Chem. Phys.*, **54**, 239 (1971).

**Table I.** Molecular Fragment Data<sup>a</sup> (au)<sup>b</sup>

	Orbital radius	Distance from carbon atom
CH <sub>4</sub> , tetrahedral, R <sub>CH</sub> = 2.05982176	ρ <sub>CH</sub> = 1.67251562 ρ <sub>C</sub> = 0.32784375	1.23379402 0.0
·CH <sub>3</sub> , planar, sp <sup>2</sup> , R <sub>CH</sub> = 1.78562447	ρ <sub>CH</sub> = 1.51399487 ρ <sub>C</sub> = 0.32682735 ρ <sub>π</sub> = 1.80394801	1.13093139 0.0 ±0.1

<sup>a</sup> See ref 10 and 11 for details of the parameter determination.  
<sup>b</sup> All distances and energies are reported throughout as unscaled quantities, using Hartree atomic units, unless otherwise stated; see H. Shull and G. G. Hall, *Nature (London)*, **184**, 1559 (1959).

Using the molecular fragment parameter data of Table I, *ab initio* SCF calculations have been carried out on benzene (I), fulvene (II), 2,3-dimethylenecyclobutene (III), trimethylenecyclopropane (IV), Dewar benzene (V), 1,3,5-*trans*-hexatriene (VI), naphthalene (VII), azulene (VIII), and fulvalene (IX). The geometries that were used are given in Table II, and the coordinate systems are shown in Figure 1.

The molecular orbitals are taken to be a linear combination of fragment FSGO's, *i.e.*

$$\varphi_i = \sum_{A=1}^P \sum_{k=1}^{N_A} c_{ki}^A G_k^A \quad (3)$$

where the  $G_k^A$ 's are the previously determined fragment orbitals, and the  $c_{ki}^A$ 's are the coefficients to be determined by the solution of the SCF equations

$$\mathbf{F}c_i = \mathbf{\Delta}c_i\epsilon \quad (4)$$

where  $\mathbf{F}$  is the Fock matrix,  $\mathbf{\Delta}$  is the overlap matrix over FSGO's, and  $\epsilon$  contains the orbital eigenvalues. For the cases not explicitly given, hydrogen atom positions have been chosen to bisect the corresponding CCC angle. In the case of benzene, the coordinates are those employed by Schulman and Moskowitz.<sup>12</sup> The SCF calculations on benzene were reported earlier,<sup>11</sup> and are summarized here only for completeness. For fulvene, the extensive VESCF studies of bond distances and angles by Brown, Burden, and Williams<sup>13</sup> were employed. For 2,3-dimethylenecyclobutene, the results of extensive geometric variations by Skancke and Skancke<sup>14</sup> were used. For trimethylenecyclopropane and 1,3,5-*trans*-hexatriene, the results of the electron diffraction studies were employed.<sup>15,16</sup> For Dewar benzene, the electron diffraction results of Cardillo and Bauer for the hexamethyl derivative<sup>17</sup> were used. Naphthalene was constructed by fusing two benzene rings, and fulvalene was constructed by fusing two fulvene rings. Azulene coordinates were taken from the crystal structure data of Hanson.<sup>18</sup>

All calculations have been carried out using double-precision arithmetic on a GE-635 computer, and the convergence criterion that was used was

$$|P_{rs}^{(i+1)} - P_{rs}^{(i)}| < 0.00002 \quad (5)$$

(12) J. M. Schulman and J. W. Moskowitz, *J. Chem. Phys.*, **43**, 3287 (1965).

(13) R. D. Brown, F. R. Burden, and G. R. Williams, *Aust. J. Chem.*, **21**, 1939 (1968).

(14) A. Skancke and P. N. Skancke, *Acta Chem. Scand.*, **22**, 12 (1968).

(15) E. A. Dorko, J. L. Hencher, and S. H. Bauer, *Tetrahedron*, **24**, 2425 (1968).

(16) M. Traetteberg, *Acta Chem. Scand.*, **22**, 628 (1968).

(17) M. J. Cardillo and S. H. Bauer, *J. Amer. Chem. Soc.*, **92**, 2399 (1970).

(18) A. W. Hanson, *Acta Crystallogr.*, **19**, 19 (1965).

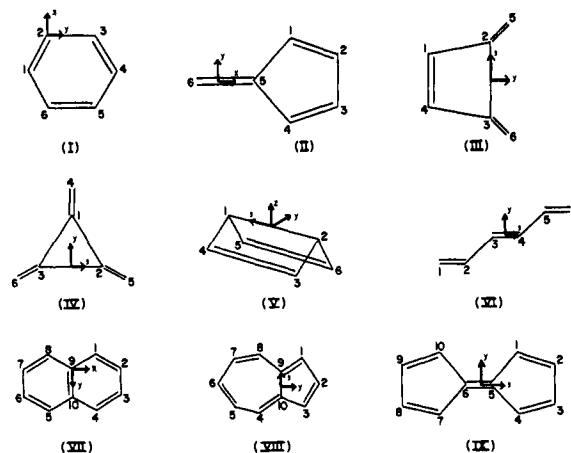


Figure 1. Numbering and coordinate systems. In all cases except Dewar benzene, the positive  $z$  axis points out of the paper.

This corresponds typically to a root-mean-square error of approximately  $10^{-7}$  or smaller.

All of the benzene (except Dewar benzene) and naphthalene isomers and hexatriene were formed by using the  $\cdot\text{CH}_3$  fragment parameters for the inner-shell carbon orbital, the  $\pi$  orbital, and the C-C bonding orbitals. The CH bonding orbital was taken from the CH<sub>4</sub> fragment. In each of these cases the position of the off-center FSGO was taken to lie along the internuclear axis, but at the same distance as found in the molecular fragment. Dewar benzene differs from the other isomers in that the bridgehead carbons were treated as sp<sup>3</sup> carbon atoms, *i.e.*, the inner-shell orbitals were taken from the CH<sub>4</sub> fragment, and no  $\pi$  orbitals were used.

## Results

Each of the benzene isomers and hexatriene were assembled using the six appropriate  $\cdot\text{CH}_3$  and CH<sub>4</sub> fragment FSGO's, and the various total energy components resulting from the SCF calculations are given in Table III. Since the scale factors in Table III are all quite similar, it is reasonable to make comparisons directly using the unscaled quantities as listed. Also, since the basis sets used for all of these molecules are identical (except for those of the bridging atoms in Dewar benzene), it is likely that error cancellations will be uniform, and that trends observed by comparisons using the energies and eigenvectors of these calculations would also be observed if more extensive basis sets were employed. Additional evidence supporting the view that the error cancellations are likely to be uniform is found in the per cent of the Hartree-Fock limit that is obtained. For all cases investigated here and previously<sup>10,11</sup> where this limit is known or can be estimated, the SCF calculations resulted in approximately 86% of the Hartree-Fock result, regardless of the particular molecule under consideration.

The molecular orbitals for the benzene isomers and hexatriene are given in Table IV. As noted in the earlier study on hydrocarbons,<sup>11</sup> the ordering of the inner-shell orbitals is not expected to be correct, since the orbitals are nearly degenerate, and a more extensive basis set is needed in order to establish the inner-shell ordering. However, in the outer-shell orbitals of usual interest in chemical reactions, previous studies<sup>11</sup> indi-

Table II. Nuclear Geometries

Molecule	Point group	Distance, Å	Angle, deg
Benzene	$D_{6h}$	$R_{CC} = 1.395$ $R_{CH} = 1.085$	$\angle C_1C_2C_3 = 120.00$
Fulvene	$C_{2v}$	$R_{C_1-C_6} = 1.460$ $R_{C_1-C_2} = 1.360$ $R_{C_2-C_3} = 1.445$ $R_{C_3-C_4} = 1.360$ $R_{CH} = 1.070$	$\angle C_4C_5C_1 = 103.00$ $\angle C_3C_1C_2 = 110.50$
2,3-Dimethylenecyclobutene	$C_{2v}$	$R_{C_1-C_2} = 1.468$ $R_{C_1-C_4} = 1.353$ $R_{C_2-C_3} = 1.350$ $R_{C_2-C_3} = 1.456$ $R_{CH} = 1.070$	$\angle C_1C_1C_2 = 92.00$ $\angle C_1C_2C_2 = 137.00$ $\angle C_1C_2C_3 = 88.0$
Trimethylenecyclopropane	$D_{3h}$	$R_{C=C} = 1.343$ $R_{C-C} = 1.453$ $R_{CH} = 1.108$	$\angle C_2C_3C_3 = 118.00$
Dewar benzene	$C_{2v}$	$R_{C_3-C_4} = 1.352$ $R_{C_1-C_2} = 1.629$ $R_{C_2-C_3} = 1.523$ $R_{CH} = 1.134$	$\angle C_3C_2C_3 = 124.50$ $\angle C_2C_1H_1 = 115.90$ $\angle C_1C_4H_4 = 120.10$ $\angle H_2C_2C_3 = 110.90$ $\angle C_1C_2C_3 = 84.78$
1,3,5- <i>trans</i> -Hexatriene	$C_{2h}$	$R_{C-C} = 1.458$ $R_{C=C} = 1.337$ $R_{C_3-C_4} = 1.368$ $R_{CH} = 1.104$	$\angle C_1C_2C_3 = 121.7$ $\angle C_2C_3C_4 = 124.4$ $\angle C_2C_1H_2 = 120.5$ $\angle C_1C_3H_4 = 115.0$ $\angle C_1C_2H_3 = 117.0$ $\angle C_1C_2C_3 = 120.0$
Naphthalene	$D_{2h}$	$R_{CC} = 1.395$ $R_{CH} = 1.085$	
Azulene	$C_{2v}$	$R_{C_9-C_1} = 1.3995$ $R_{C_1-C_2} = 1.3925$ $R_{C_8-C_9} = 1.3910$ $R_{C_7-C_8} = 1.3975$ $R_{C_4-C_7} = 1.3940$ $R_{C_9-C_{10}} = 1.4970$ $R_{CH} = 1.070$ $R_{C_5-C_6} = 1.3600$	$\angle C_{10}C_9C_1 = 106.315$ $\angle C_{10}C_9C_8 = 127.410$ $\angle C_9C_8C_7 = 128.810$ $\angle C_8C_7C_6 = 129.055$ $\angle C_9C_1C_2 = 108.615$
Fulvalene	$D_{2h}$		<i>a</i>

<sup>a</sup> Same ring geometry as fulvene.

Table III. Total Energy Components for Benzene Isomers and Hexatriene

Molecule	Energy components <sup>a</sup>				Total energy	$\sum_i^{\text{occ}} \epsilon_i(\pi)$	Scale factor <sup>b</sup>
	Kinetic	Electron-nuclear	Electron-electron	Nuclear-nuclear			
Benzene	182.99669075	-855.31125641	271.12976837	203.83065414	-197.35413742	-0.82536	1.03922869
Fulvene	183.08301544	-850.68164825	269.10876846	201.25369644	-197.23616791	-0.75075	1.03865227
2,3-Dimethylene-cyclobutene	183.26167107	-843.94989777	265.68368530	197.84906387	-197.15546608	-0.76913	1.03790699
Trimethylene-cyclopropane	183.54508591	-824.18850708	256.02054214	187.60945892	-197.01341438	-0.76124	1.03668942
Dewar benzene	183.38595009	-864.31048584	278.18297195	205.95754433	-196.78401184		1.03652968
Hexatriene	184.52228737	-836.95754242	261.23413467	193.05647087	-198.14464188	-0.74308	1.03691249

<sup>a</sup> All energies and distances are reported throughout as unscaled quantities, using hartree atomic units, unless otherwise stated; see footnote *b*, Table I. <sup>b</sup> See P. O. Löwdin, *J. Mol. Spectrosc.*, **3**, 46 (1959), for the definition of the scale factor.

cate that the ordering of the various symmetries is expected to be correctly given.

The total energy components for the naphthalene isomers, formed from ten  $\cdot\text{CH}_3$  and  $\text{CH}_4$  fragment FSGO's, are summarized in Table V. As in the case of the benzene isomers, the scale factors for the naphthalene isomers are quite similar, so comparisons will be made directly using the unscaled energies given in Table V.

The molecular orbitals and associated symmetries that result from these calculations are given in Table VI. As in the case of the benzene isomers, it is not expected that the molecular orbitals will be well described for the inner-shell electrons. Consequently, most comments will be directed toward the non-inner-shell "valence" orbitals.

## Discussion

**A. Comparisons with Other Calculations.** It is important first to calibrate these results, in order to ascertain the accuracy to be expected. Toward this end, it is instructive to examine where similarities and discrepancies occur between these calculations and more extensive basis set calculations.

In the case of benzene isomers and hexatriene, several comparisons are possible. For benzene itself, it was found in a previous study<sup>11</sup> that the ordering of the molecular orbitals that results using the fragment orbitals described in Table I (a total of 36 FSGO's) is precisely the same for the "valence" orbitals as obtained by using a considerably larger basis set<sup>19</sup> (a total of 222

(19) R. J. Buenker, J. L. Whitten, and J. D. Petke, *J. Chem. Phys.*, **49**, 2261 (1968).

Table IV. Molecular Orbitals for Benzene Isomers and Hexatriene

Benzene ( $D_{6h}$ )		Fulvene ( $C_{2v}$ )		2,3-Dimethylene- cyclobutene ( $C_{2v}$ )		Trimethylene- cyclopropene ( $C_{2v}$ )		Dewar benzene ( $C_{2v}$ )		Hexatriene ( $C_{2h}$ )	
MO	$-\epsilon$	MO	$-\epsilon$	MO	$-\epsilon$	MO	$-\epsilon$	MO	$-\epsilon$	MO	$-\epsilon$
1b <sub>1u</sub>	9.4535	1a <sub>1</sub>	9.4265	1b <sub>2</sub>	9.4419	1e'	9.3986	1b <sub>2</sub>	9.3265	1b <sub>u</sub>	9.4136
1e <sub>2g</sub>	9.4252	1b <sub>2</sub>	9.4253	2b <sub>2</sub>	9.4027	1e'	9.3986	1a <sub>2</sub>	9.3203	1a <sub>g</sub>	9.4122
1e <sub>2g</sub>	9.4252	2a <sub>1</sub>	9.4067	1a <sub>1</sub>	9.3947	1a <sub>1</sub> '	9.3919	1a <sub>1</sub>	9.3207	2b <sub>u</sub>	9.4016
1e <sub>1u</sub>	9.3517	2b <sub>2</sub>	9.3322	2a <sub>1</sub>	9.3507	2e'	0.3355	1b <sub>1</sub>	9.2264	2a <sub>g</sub>	9.3442
1e <sub>1u</sub>	9.3517	3a <sub>1</sub>	9.3291	3b <sub>2</sub>	9.3294	2e'	9.3355	2b <sub>2</sub>	9.1974	3b <sub>u</sub>	9.2833
1a <sub>1g</sub>	9.2684	4a <sub>1</sub>	9.2419	3a <sub>1</sub>	9.2431	2a <sub>1</sub> '	9.2370	2a <sub>1</sub>	9.0790	3a <sub>g</sub>	9.2479
2a <sub>1g</sub>	1.0874	5a <sub>1</sub>	1.0840	4a <sub>1</sub>	1.1065	3a <sub>1</sub> '	1.0518	3a <sub>1</sub>	0.9925	4a <sub>g</sub>	1.0261
2e <sub>1u</sub>	0.9686	6a <sub>1</sub>	0.9705	4b <sub>2</sub>	0.9397	3e'	0.9371	2b <sub>1</sub>	0.8751	4b <sub>u</sub>	0.9870
2e <sub>1u</sub>	0.9686	3b <sub>2</sub>	0.8994	5a <sub>1</sub>	0.9358	3e'	0.9371	3b <sub>2</sub>	0.7437	5a <sub>g</sub>	0.9130
2e <sub>2g</sub>	0.7605	7a <sub>1</sub>	0.8042	6a <sub>1</sub>	0.7469	4a <sub>1</sub> '	0.8458	4a <sub>1</sub>	0.6676	5b <sub>u</sub>	0.7604
2e <sub>2g</sub>	0.7605	4b <sub>2</sub>	0.6570	5b <sub>2</sub>	0.7442	4e'	0.5947	2a <sub>2</sub>	0.6389	6a <sub>g</sub>	0.6732
3a <sub>1g</sub>	0.5848	8a <sub>1</sub>	0.6371	6b <sub>2</sub>	0.5788	4e'	0.5947	4b <sub>2</sub>	0.5155	6b <sub>u</sub>	0.6581
2b <sub>1u</sub>	0.5566	9a <sub>1</sub>	0.5748	7a <sub>1</sub>	0.5627	5a <sub>1</sub> '	0.5277	5a <sub>1</sub>	0.4962	7a <sub>g</sub>	0.5252
1b <sub>2u</sub>	0.5031	5b <sub>2</sub>	0.5034	8a <sub>1</sub>	0.4993	5e'	0.4715	3b <sub>1</sub>	0.4724	7b <sub>u</sub>	0.5028
3e <sub>1u</sub>	0.4783	10a <sub>1</sub>	0.4408	7b <sub>2</sub>	0.4726	5e'	0.4715	6a <sub>1</sub>	0.3607	8b <sub>u</sub>	0.4705
3e <sub>1u</sub>	0.4783	6b <sub>2</sub>	0.4389	9a <sub>1</sub>	0.4211	1a <sub>2</sub> '	0.4645	4b <sub>1</sub>	0.3139	8a <sub>g</sub>	0.4402
1a <sub>2u</sub> ( $\pi$ )	0.3915	11a <sub>1</sub>	0.4030	1b <sub>1</sub> ( $\pi$ )	0.3837	1a <sub>2</sub> ' ( $\pi$ )	0.3587	5b <sub>2</sub>	0.2530	9a <sub>g</sub>	0.4051
3e <sub>2g</sub>	0.3900	1b <sub>1</sub> ( $\pi$ )	0.3758	8b <sub>2</sub>	0.3787	6e'	0.2734	3a <sub>2</sub>	0.2047	9b <sub>u</sub>	0.3993
3e <sub>2g</sub>	0.3900	7b <sub>2</sub>	0.3652	10a <sub>1</sub>	0.3424	6e'	0.2734	7a <sub>1</sub>	0.1758	10a <sub>g</sub>	0.3767
1e <sub>1g</sub> ( $\pi$ )	0.2169	2b <sub>1</sub> ( $\pi$ )	0.2156	1a <sub>2</sub> ( $\pi$ )	0.2050	1e'' ( $\pi$ )	0.2012	5b <sub>1</sub>	0.1345	1a <sub>u</sub> ( $\pi$ )	0.3320
1e <sub>1g</sub> ( $\pi$ )	0.2169	1a <sub>2</sub> ( $\pi$ )	0.1594	2b <sub>1</sub> ( $\pi$ )	0.1803	1e'' ( $\pi$ )	0.2012	8a <sub>1</sub>	0.0546	1b <sub>g</sub> ( $\pi$ )	0.2631
										2a <sub>u</sub> ( $\pi$ )	0.1480
1e <sub>2u</sub> ( $\pi^*$ )	-0.2913	3b <sub>1</sub> ( $\pi^*$ )	-0.2016	2a <sub>2</sub> ( $\pi^*$ )	-0.2654	2a <sub>2</sub> ' ( $\pi^*$ )	-0.2041	6b <sub>2</sub>	-0.3790	2b <sub>g</sub> ( $\pi^*$ )	-0.2389
1e <sub>2u</sub> ( $\pi^*$ )	-0.2913	2a <sub>2</sub> ( $\pi^*$ )	-0.4293	3b <sub>1</sub> ( $\pi^*$ )	-0.3023	2e'' ( $\pi^*$ )	-0.4819	4a <sub>2</sub>	-0.4223	3a <sub>u</sub> ( $\pi^*$ )	-0.3782
1b <sub>2g</sub> ( $\pi^*$ )	-0.5441	4b <sub>1</sub> ( $\pi^*$ )	-0.4811	3a <sub>2</sub> ( $\pi^*$ )	-0.5572	2e'' ( $\pi^*$ )	-0.4819	7b <sub>2</sub>	-0.7993	3b <sub>g</sub> ( $\pi^*$ )	-0.4835
4e <sub>2g</sub>	-0.9999	8b <sub>2</sub>	-0.9143	9b <sub>2</sub>	-0.8569	2a <sub>2</sub> '	-0.7549	9a <sub>1</sub>	-0.9765	10b <sub>u</sub>	-0.8496
4e <sub>2g</sub>	-0.9999	12a <sub>1</sub>	-0.9312	11a <sub>1</sub>	-0.8765	7e'	-0.9752	6b <sub>1</sub>	-1.0545	11b <sub>u</sub>	-0.9810
4e <sub>2u</sub>	-1.0406	9b <sub>2</sub>	-0.9819	10b <sub>2</sub>	-0.9924	7e'	-0.9752	5a <sub>2</sub>	-1.0758	11a <sub>g</sub>	-1.0154

Table V. Total Energy Components<sup>a</sup> for Naphthalene Isomers

Energy component	Naphthalene	Azulene	Fulvalene
Kinetic	303.35664749	303.51782608	303.47954559
Electron-nuclear	-1667.78880310	-1653.77546692	-1631.76205444
Electron-electron	574.26500702	567.07115936	556.76239777
Nuclear-nuclear	462.05744553	455.22081757	443.62667084
Total $E$	-328.10969543	-327.96566010	-327.89342499
$\Sigma_i^{\text{occ}} \epsilon_i(\pi)$	-1.3770	-1.3690	-1.2631
Scale factor	1.04079859	1.04027413	1.04022326

<sup>a</sup> All energies are reported as unscaled quantities.

Gaussians). Furthermore, it was shown that the remaining energetic deficiency of the current description was primarily in the inner-shell-orbital description. It is interesting to note that another investigation,<sup>20</sup> using a larger basis set than the one employed here, did not predict the correct ordering of the valence molecular orbitals, with the ordering of the 1b<sub>2u</sub> and 2b<sub>1u</sub> orbitals switched from that found by Buenker, Whitten, and Petke.<sup>19</sup> Consequently, increasing the size of the basis set does not guarantee more satisfactory results, unless the balance of the basis set can be maintained.

For the case of fulvene, direct comparisons with more extensive calculations are also possible. For a basis set of 174 Gaussians, it was found<sup>21</sup> that all of the filled non-inner-shell molecular orbitals are in exactly the same order as the order given in Table IV. This includes the ordering of most virtual, as well as filled, orbitals. As in the case of benzene, the basis set employed by Praud, *et al.*,<sup>20</sup> does not produce an ordering of molecular orbitals in agreement with Peyerimhoff<sup>21</sup> and the current studies, with the ordering of the 1b<sub>1</sub> and 11a<sub>1</sub> orbitals

switched. For the case of 2,3-dimethylenecyclobutene, the ordering is the same as that observed by Praud, *et al.*,<sup>20</sup> except for the 6b<sub>2</sub>, 7a pair, which is switched.

It is also possible to make direct comparisons with more extensive basis-set calculations for the naphthalene and azulene molecules.<sup>22</sup> For naphthalene, only one switch in ordering is observed in the filled non-inner-shell orbitals. This occurs in the case of the 9a<sub>g</sub> and 6b<sub>1g</sub> orbitals, whose energies are very nearly degenerate. In the case of the virtual orbitals, only the ordering of the very high-lying 8b<sub>3u</sub> and 8b<sub>2u</sub> orbitals is interchanged from that of Buenker and Peyerimhoff.<sup>22</sup> Considering the large number of symmetry species that are present (eight irreducible representations), the agreement is remarkably complete. Furthermore, a direct comparison of the molecular orbital coefficients of the  $\pi$  orbitals also shows striking similarities, as indicated in Table VII.

For the case of azulene, the ordering of the molecular orbitals is again found to be identical with that found by Buenker and Peyerimhoff<sup>22</sup> for the chemically inter-

(20) L. Praud, P. Millie, and G. Berthier, *Theor. Chim. Acta*, 11, 169 (1968).

(21) S. D. Peyerimhoff, private communication.

(22) R. J. Buenker and S. D. Peyerimhoff, *Chem. Phys. Lett.*, 3, 37 (1969).

Table VI. Molecular Orbitals for Naphthalene Isomers

Naphthalene ( $D_{2h}$ ) MO	Azulene ( $C_{2v}$ ) MO		Fulvalene ( $D_{2h}$ ) MO		
	$-\epsilon$		$-\epsilon$	$-\epsilon$	
1b <sub>2u</sub>	9.4516	1a <sub>1</sub>	9.4724	1b <sub>1g</sub>	9.4321
1b <sub>1g</sub>	9.4497	1b <sub>2</sub>	9.4608	1b <sub>2u</sub>	9.4311
1a <sub>g</sub>	9.4295	2a <sub>1</sub>	9.4360	1b <sub>3u</sub>	9.4247
1b <sub>3u</sub>	9.4232	3a <sub>1</sub>	9.4302	1a <sub>g</sub>	9.4182
2b <sub>2u</sub>	9.4172	2b <sub>2</sub>	9.4171	2b <sub>3u</sub>	9.3918
2b <sub>1g</sub>	9.3817	3b <sub>2</sub>	9.3896	2a <sub>g</sub>	9.3478
2a <sub>g</sub>	9.3607	4a <sub>1</sub>	9.3698	2b <sub>1g</sub>	0.3432
3b <sub>2u</sub>	9.3396	4b <sub>2</sub>	9.3361	2b <sub>3u</sub>	9.3377
2b <sub>3u</sub>	9.3083	5a <sub>1</sub>	9.3247	3b <sub>3u</sub>	9.2752
3a <sub>g</sub>	9.2666	6a <sub>1</sub>	9.2692	3a <sub>g</sub>	9.2473
4a <sub>g</sub>	1.1188	7a <sub>1</sub>	1.1109	4a <sub>g</sub>	1.1020
3b <sub>3u</sub>	1.0538	8a <sub>1</sub>	1.0650	4b <sub>3u</sub>	1.0781
4b <sub>2u</sub>	1.0040	15b <sub>2</sub>	1.0283	5a <sub>g</sub>	0.9581
5a <sub>g</sub>	0.9654	9a <sub>1</sub>	0.9617	3b <sub>2u</sub>	0.9112
3b <sub>1g</sub>	0.9246	6b <sub>1</sub>	0.9099	3b <sub>1g</sub>	0.8986
4b <sub>3u</sub>	0.7997	10a <sub>1</sub>	0.8282	5b <sub>3u</sub>	0.8319
5b <sub>2u</sub>	0.7687	7b <sub>2</sub>	0.7952	6a <sub>g</sub>	0.7137
6a <sub>g</sub>	0.7453	11a <sub>1</sub>	0.6700	4b <sub>2u</sub>	0.6719
4b <sub>1g</sub>	0.6255	12a <sub>1</sub>	0.6506	4b <sub>1g</sub>	0.6455
5b <sub>3u</sub>	0.5803	8b <sub>2</sub>	0.6256	6b <sub>3u</sub>	0.6190
7a <sub>g</sub>	0.5561	13a <sub>1</sub>	0.5638	7a <sub>g</sub>	0.6033
6b <sub>2u</sub>	0.5255	14a <sub>1</sub>	0.5270	7b <sub>3u</sub>	0.5241
8a <sub>g</sub>	0.5009	9b <sub>2</sub>	0.5121	5b <sub>2u</sub>	0.4486
5b <sub>1g</sub>	0.4725	10b <sub>2</sub>	0.4684	5b <sub>1g</sub>	0.4400
6b <sub>3u</sub>	0.4686	15a <sub>1</sub>	0.4497	8a <sub>g</sub>	0.4301
7b <sub>2u</sub>	0.4522	11b <sub>2</sub>	0.4249	6b <sub>2u</sub>	0.4190
1b <sub>1u</sub> ( $\pi$ )	0.4261	16a <sub>1</sub>	0.4238	9a <sub>g</sub>	0.4081
7b <sub>3u</sub>	0.4234	1b <sub>1</sub> ( $\pi$ )	0.4192	8b <sub>3u</sub>	0.4062
9a <sub>g</sub>	0.3781	12b <sub>2</sub>	0.4084	1b <sub>1u</sub> ( $\pi$ )	0.4026
6b <sub>1g</sub>	0.3755	17a <sub>1</sub>	0.3653	6b <sub>1g</sub>	0.3574
1b <sub>2g</sub> ( $\pi$ )	0.3304	2b <sub>1</sub> ( $\pi$ )	0.3410	1b <sub>2g</sub> ( $\pi$ )	0.3461
1b <sub>3g</sub> ( $\pi$ )	0.2658	1a <sub>2</sub> ( $\pi$ )	0.2982	2b <sub>1u</sub> ( $\pi$ )	0.1862
2b <sub>1u</sub> ( $\pi$ )	0.2002	3b <sub>1</sub> ( $\pi$ )	0.1808	1b <sub>3g</sub> ( $\pi$ )	0.1672
1a <sub>u</sub> ( $\pi$ )	0.1545	2a <sub>2</sub> ( $\pi$ )	0.2198	1a <sub>u</sub> ( $\pi$ )	0.1610
2b <sub>2g</sub> ( $\pi^*$ )	-0.2253	4b <sub>1</sub> ( $\pi^*$ )	-0.1822	2b <sub>2g</sub> ( $\pi^*$ )	-0.1528
2b <sub>3g</sub> ( $\pi^*$ )	-0.2845	3a <sub>2</sub> ( $\pi^*$ )	-0.2137	3b <sub>1u</sub> ( $\pi^*$ )	-0.3429
3b <sub>1u</sub> ( $\pi^*$ )	-0.3502	5b <sub>1</sub> ( $\pi^*$ )	-0.4216	2b <sub>3g</sub> ( $\pi^*$ )	-0.4198
2a <sub>u</sub> ( $\pi^*$ )	-0.4475	6b <sub>1</sub> ( $\pi^*$ )	-0.4974	2a <sub>u</sub> ( $\pi^*$ )	-0.4294
3b <sub>3g</sub> ( $\pi^*$ )	-0.6310	4a <sub>2</sub> ( $\pi^*$ )	-0.5368	3b <sub>2g</sub> ( $\pi^*$ )	-0.5521
8b <sub>3u</sub>	-0.7721	18a <sub>1</sub>	-0.8153	7b <sub>2u</sub>	-0.7633
8b <sub>2u</sub>	-0.9363	13b <sub>2</sub>	-0.8507	9b <sub>3u</sub>	-0.8689

esting valence orbitals, with the exception of the near-degenerate 11b<sub>2</sub> and 16a<sub>1</sub> orbitals and the highest virtual orbitals (18a<sub>1</sub> and 13b<sub>2</sub>). Detailed comparison of the magnitudes of the individual coefficients also indicates essentially complete agreement, with the largest error being 0.11 in the 4a<sub>2</sub> orbital, the highest virtual orbital.

Consequently, it is clear that, for the molecules where direct comparisons with more extensive calculations are possible, the electronic structure as predicted by the current procedure is essentially identical, even to the point of detailed comparisons of molecular orbital coefficients. Therefore, comparisons of electronic structure differences in hydrocarbons to be made below are expected to yield reliable and useful measures of the differences in the actual electronic structure of the various molecules. The only exceptions to be expected are for nearly degenerate or very high-lying orbitals, and they will be noted accordingly.

**B. Relative Stabilities.** Of considerable interest is the relative stability of the various isomers of benzene and naphthalene. The appropriate energy differences for these isomers are presented in Table VIII. It should be pointed out that the absolute magnitude of these energy differences is expected to be exaggerated, for

Table VII.  $\pi$ -Orbital Comparisons for Naphthalene

MO	—MO coefficients—		Atoms <sup>a</sup>
	This work	Ref 22	
1b <sub>1u</sub> ( $\pi$ )	0.31643	0.31307	9 + 10
	0.21762	0.21139	1 + 4 + 5 + 8
	0.17914	0.17409	2 + 3 + 6 + 7
1b <sub>2g</sub> ( $\pi$ )	0.21681	0.22442	1 + 4 - 5 - 8
	0.33256	0.32958	2 + 3 - 6 - 7
	0.27544	0.27703	9 - 10
1b <sub>3g</sub> ( $\pi$ )	0.33897	0.34500	1 - 4 - 5 + 8
	0.15612	0.17837	2 - 3 - 6 + 7
	0.38484	0.43450	9 + 10
2b <sub>1u</sub> ( $\pi$ )	0.00477	-0.01850	1 + 4 + 5 + 8
	-0.35042	-0.35344	2 + 3 + 6 + 7
	0.39231	0.40142	1 - 4 + 5 - 8
1a <sub>u</sub> ( $\pi$ )	0.25709	0.27262	2 - 3 + 6 - 7
	-0.49289	-0.49692	1 + 4 - 5 - 8
	0.31478	0.33223	2 + 3 - 6 - 7
2b <sub>2g</sub> ( $\pi^*$ )	0.53529	0.57492	9 - 10
	0.00619	0.00268	1 - 4 - 5 + 8
	-0.51041	-0.47500	2 - 3 - 6 + 7
3b <sub>1u</sub> ( $\pi^*$ )	-0.47722	-0.43572	9 + 10
	0.53877	0.51696	1 + 4 + 5 + 8
	-0.23517	-0.24960	2 + 3 + 6 + 7
2a <sub>u</sub> ( $\pi^*$ )	0.39578	0.38763	1 - 4 + 5 - 8
	-0.61809	-0.57368	2 - 3 + 6 - 7
	-0.84964	-0.67857	9 - 10
3b <sub>3g</sub> ( $\pi^*$ )	0.53395	0.45016	1 - 4 - 5 + 8
	-0.41380	-0.38068	2 - 3 - 6 + 7

<sup>a</sup> Refers to the atom on which the  $\pi$  orbital is located (see Figure 1).

Table VIII. Relative Stability of Benzene and Naphthalene Isomers and Hexatriene

Molecule	Energy difference, kcal/mol	Molecule	Energy difference, kcal/mol
Benzene		Naphthalene	
Fulvene	74		
2,3-Dimethylene-cyclobutene	125	Azulene	90
Trimethylene-cyclopropane	213	Fulvalene	135
Dewar benzene	357		
Hexatriene	-496		

several reasons. First, a similar magnification of energy differences was observed in studies of various hydrocarbon rotomers.<sup>11</sup> Also, the correlation error for decreasing size ring systems will be likely to increase, further exaggerating the effect. Nevertheless, several interesting observations can be made concerning the relative stabilities.

First, it is obvious that hexatriene itself (which is neither isoelectronic nor isonuclear to benzene) is not a useful reference molecule, if total energies are used for the comparisons. Correction for the extra hydrogen atoms does not alter this observation. However, the individual molecular orbital structure is more promising in this regard, as will be discussed later.

Although the magnitude of these energy differences may be reduced by more extensive basis-set calculations, it appears unlikely that the relative ordering of the various isomers will be altered. For comparison purposes, it might be noted that more extensive SCF calculations<sup>21</sup> indicate that benzene is more stable than fulvene by 40.2 kcal/mol, and that naphthalene is more stable than azulene by 50.3 kcal/mol.<sup>22</sup> These very extensive calculations also overestimate the stability

difference,<sup>23</sup> emphasizing the difficulty of predicting absolute differences in energies without using configuration interaction techniques.

It should be noted also that the results of semiempirical calculations using both the MINDO technique and a  $\pi$  procedure<sup>24</sup> are not in agreement with the results of the current studies. In particular, the MINDO procedure predicts a relative ordering benzene, fulvene, 2,3-dimethylenecyclobutene, Dewar benzene, trimethylenecyclopropane, while the  $\pi$  procedure predicts a relative ordering benzene, fulvene, trimethylenecyclopropane, 2,3-dimethylenecyclobutene. Since the current calculations (where comparisons are possible) have been found to give electronic structure predictions in essentially complete agreement with the most accurate theoretical calculations available, it is believed that the relative order of stabilities given by the current studies is that expected to be verified experimentally. Consequently, the  $\sigma$  electrons in these systems appear to play a very important role, both with respect to the overall stability and the stability of the  $\pi$  orbitals, and should be explicitly considered if comparisons are to be made.<sup>25</sup>

The ionization potentials, as estimated *via* Koopmans' theorem,<sup>26</sup> are summarized in Table IX. Con-

Table IX. Ionization Potentials of Benzene and Naphthalene Isomers and Hexatriene

Molecule	IP, eV	
	Calcd	Exptl
Benzene	5.90	9.241 <sup>a</sup>
Fulvene	4.34	
2,3-Dimethylenecyclobutene	4.91	
Trimethylenecyclopropane	5.48	
Dewar benzene	1.49	
Hexatriene	4.03	
Naphthalene	4.20	8.12 <sup>b</sup>
Azulene	3.53	7.41 <sup>c</sup>
Fulvalene	4.38	

<sup>a</sup> A. J. C. Nicholson, *J. Chem. Phys.*, **43**, 1171 (1965). <sup>b</sup> K. Watanabe, T. Nakayama, and J. Mottle, *J. Quant. Spectrosc., Radiat. Transfer*, **2**, 369 (1962). <sup>c</sup> T. Kitagawa, H. Inokuchi, and K. Kodera, *J. Mol. Spectrosc.*, **21**, 267 (1966).

sistent with earlier observations,<sup>11</sup> the values are found to be too low when compared with more extensive calculations.<sup>19-22</sup> However, the trends appear to be correctly indicated, as for other properties.

It might be noted that the observation of Buenker and Peyerimhoff<sup>22</sup> concerning comparison of inner-shell orbital energies to extract differences in valence-charge distribution is not borne out in these studies. The inner-shell orbitals resulting from the current studies are not localized, so the identification of a given orbital with a particular nucleus (or a set of symmetry-related nuclei) is not possible without performing a separate localization procedure.<sup>27</sup>

(23) E. Perrottet, W. Taub, and E. Briner, *Helv. Chim. Acta*, **23**, 1260 (1940).

(24) N. C. Baird and M. J. S. Dewar, *J. Amer. Chem. Soc.*, **91**, 352 (1969).

(25) This observation has also been made for other hydrocarbons; see, for example, R. J. Buenker and S. D. Peyerimhoff, *J. Chem. Phys.*, **48**, 354 (1968).

(26) T. A. Koopmans, *Physica*, **1**, 104 (1933).

(27) See, for example, S. F. Boys, *Rev. Mod. Phys.*, **32**, 296 (1960); C. Edmiston and K. Ruedenberg, *ibid.*, **34**, 457 (1963); V. Magnasco and A. Perico, *J. Chem. Phys.*, **47**, 971 (1967).

**C. Molecular Orbital Structure and Chemical Characteristics.** Several comments regarding the nature and ordering of the various molecular orbitals are also appropriate. From Tables IV and VI, the first observation of interest is the considerable interspersing of  $\sigma$  and  $\pi$  orbitals. Only hexatriene has all of the  $\pi$  orbitals grouped together as the uppermost filled orbitals. The importance of this is emphasized by the fact that the interspersing is not uniform in the various isomers, *e.g.*, three  $\sigma$  orbitals appear between the  $1b_{1u}$  ( $\pi$ ) and the  $1b_{2g}$  ( $\pi$ ) orbitals in naphthalene, and only one  $\sigma$  orbital appears between the same symmetry orbitals in fulvalene.

In the case of Dewar benzene, the lack of planarity destroys any identifiable  $\pi$  orbitals, and the electronic structure is considerably different from that of the other isomers. For example, the  $b_1$  and  $a_2$  orbitals, which appear as  $\pi$  orbitals in the other isomers of  $C_{2v}$  symmetry, appear interspersed within the entire set of orbitals, and are not all grouped near the top of the filled orbitals. Indeed,  $1a_2$  and  $1b_1$  appear as *inner-shell* orbitals.

In the  $C_{10}H_8$  isomer series, both naphthalene and fulvalene have  $D_{2h}$  symmetry and show several striking similarities in their electronic structure, as seen by examination of Table VI. For example, the number and kind of molecular orbitals that are filled are quite similar, differing only in the number of  $b_{2u}$  and  $b_{3u}$  orbitals. Furthermore, the highest occupied orbital is  $1a_u$  in both cases, and the ionization potentials (Table IX) are quite similar. In addition, the detailed orbital shapes, including FSGO molecular orbital coefficients, are quite similar, as indicated in Figure 2. As seen in the figure, carbons 5 and 6 in fulvalene play a role entirely analogous to that of carbons 9 and 10 in naphthalene, while carbons 2, 3, 8, and 9 in fulvalene correspond to carbons 2, 3, 6, and 7 in naphthalene, and carbons 1, 4, 7, and 10 in fulvalene correspond to carbons 1, 4, 5, and 8 in naphthalene. Similar observations also apply to the shapes of the other filled  $\pi$  molecular orbitals of the same symmetry in naphthalene and fulvalene.

As far as the chemical reactivity of a molecule is determined by the nature of the highest occupied and lowest unoccupied molecular orbitals,<sup>28</sup> naphthalene and fulvalene should possess similar reaction characteristics. Azulene, on the other hand, does not contain a node at carbons 9 and 10 in its highest occupied molecular orbital (as is present at carbons 9 and 10 in naphthalene and carbons 5 and 6 in fulvalene), even though a total of two nodes is still present. Apparently the position of the orbital nodes remains relatively stationary (relative to the  $\pm x$  coordinates), and is relatively insensitive to changes in the atomic geometry. Consequently, the susceptibility to electrophilic attack at carbons 9 and 10 in azulene is expected to be greater than at carbons 9 and 10 in naphthalene and carbons 5 and 6 in fulvalene.

The other molecular orbital coefficients in Figure 2 also reveal interesting points concerning the chemistry of these species. In naphthalene, the magnitude of the highest occupied molecular orbital coefficients (a measure of the electron density at various atoms in this molecular orbital) is seen to mimic the chemical reactivity of naphthalene,<sup>29</sup> where electrophilic attack would

(28) For many examples of this notion, see R. Hoffmann and R. B. Woodward, *Science*, **167**, 825 (1970), and earlier papers.

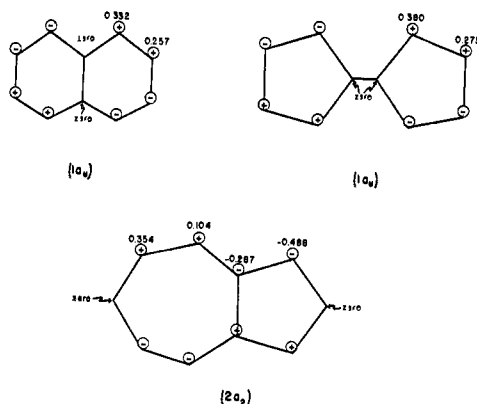


Figure 2. Comparisons of the highest occupied orbitals in the naphthalene isomer series. The magnitude of the coefficient of the FSGO on the respective atoms is also given.

be expected to be most favorable at carbons 1, 4, 5, and 8, and nucleophilic attack would be favored at atoms 9 and 10. Similarly, fulvalene would be expected to be susceptible to electrophilic attack at atoms 1, 4, 7, and 10, and most susceptible to nucleophilic attack at atoms 5 and 6. Azulene, on the other hand, does not have reaction characteristics that can be rationalized by examination of only the highest occupied molecular orbital coefficients, and a larger set of molecular orbitals must be considered. Further investigations are currently in progress to clarify this possibility.

Although it is likely that the ordering of the various molecular orbitals is correctly revealed by these calculations, there are several places at which accidental degeneracies approximately occur, in which the ordering may be altered by more extensive basis set calculations. In addition to those already noted in direct comparisons, one should include the  $9a_g, 9b_u$  pair in hexatriene and the  $5b_{2u}, 5b_{1g}$  pair, the  $9a_g, 8b_{3u}, 1b_{1u}$  ( $\pi$ ) triad, and the  $1b_{3g}$  ( $\pi$ ),  $1a_u$  ( $\pi$ ) pair in fulvalene. The ordering of the latter pair is of particular interest, since these orbitals represent the highest occupied orbitals. However, the overall reactivity at various carbon positions in fulvalene is not expected to be affected greatly by the ordering of the  $1b_{3g}$  ( $\pi$ ),  $1a_u$  ( $\pi$ ) pair, since both orbitals have nodes at the same atoms (5 and 6 in fulvalene and 9 and 10 in naphthalene) and coefficients whose magnitudes are approximately the same in both molecular orbitals at the various atoms.<sup>30</sup>

**D. Aromaticity Considerations.** The determination of the electronic structure of the various benzene and naphthalene isomers also can be profitably examined with respect to the concept of aromaticity. This notion, useful in rationalizing the differences in stability of various ring compounds from their open-chain analogs,<sup>31</sup> is connected closely with the presence of delocalized  $\pi$  orbitals in the ring compounds, which impart added stability ("aromaticity") that would not be present if these orbitals were localized. Consequently, some of the currently used procedures<sup>2</sup> for quantification of this notion are based upon a comparison of a molecule of

interest with a corresponding hypothetical molecule having all "localized"  $\pi$  orbitals. However, it seems that, if properties that are intrinsically *real* chemical characteristics of molecules are to be compared, it is highly desirable to avoid comparisons based upon *unreal* reference molecules.

It appears from the data presented here that the notion of aromaticity can be rationalized without the introduction of hypothetical reference molecules. Instead, the electronic structure as given by the wave functions for the actual molecules can be used to make the desired comparisons. In particular, the occupied  $\pi$  orbitals resulting from these calculations exhibit characteristics that provide an acceptable framework for both qualitative and quantitative discussions of aromaticity. It must be emphasized at the outset, however, that the relative energies of the  $\pi$  orbitals are strongly dependent upon the particular  $\sigma$  environment that is present. Consequently, explicit consideration of the  $\sigma$  electrons must be included in order to obtain reliable  $\pi$ -orbital determinations. It should also be noted that, since aromaticity is a concept that is concerned only with  $\pi$  orbitals, the aromaticities of a set of molecules need not necessarily parallel the ordering of overall stability of the molecules.

Considering hexatriene and the benzene isomers, we note first that the filled  $\pi$  orbitals in hexatriene all lie above the filled  $\sigma$  orbitals and have not been stabilized by the particular  $\sigma$ -electron and nuclear environment to any extent more than expected from particle-in-a-box considerations.<sup>32</sup> However, in fulvalene, the lowest  $\pi$  orbital has been sufficiently stabilized by this particular environment to exchange its position relative to the highest  $\sigma$  orbital ( $7b_2$ ). In 2,3-dimethylenecyclobutene, trimethylenecyclopropane, and benzene itself, the  $\pi$  orbitals are further stabilized, with a pair of  $\sigma$  orbitals separating the lowest  $\pi$  orbital from the other filled  $\pi$  orbitals. Consequently, a qualitative understanding of the aromaticity of these molecules can be obtained from simple examination of the relative ordering of molecular orbitals.

More quantitatively, the amount of lowering of a given  $\pi$  orbital can be estimated by its orbital energy. Since all filled  $\pi$  orbitals contribute to the aromaticity of a given molecule, the sum of the filled  $\pi$ -orbital energies can be used to provide a simple measure of the extent of energy lowering. In Table III, these quantities are given for hexatriene and the benzene isomers. As expected, they parallel the qualitative observations made above, with benzene and hexatriene exhibiting the aromaticity extremes, and the other isomers falling between these two molecules. In addition, the quantitative measure shows that the three-, four-, and five-membered rings all exhibit aromaticities (*i.e.*,  $\sum_i^{\text{occ}} \epsilon_i(\pi)$ ) closer to hexatriene than to benzene, in agreement with their chemical behavior.<sup>13, 15, 23, 34</sup> Thus, the overall description that emerges is that the three-, four-, and five-membered ring isomers are primarily *nonaromatic*, with only benzene having considerable aromatic character.

(29) J. D. Roberts and M. C. Caserio, "Basic Principles of Organic Chemistry," W. A. Benjamin, New York, N. Y., 1965, p 812.

(30) For example, the  $\pi$  FSGO on carbon 4 in fulvalene has a coefficient of  $\pm 0.384$  in the  $1b_{3g}$  ( $\pi$ ) orbital, and a coefficient of  $-0.380$  in the  $1a_u$  ( $\pi$ ) orbital.

(31) See, for example, M. J. S. Dewar, *Tetrahedron Suppl.*, No. 8, 75 (1966).

(32) R. L. Flurry, Jr., "Molecular Orbital Theories of Bonding in Organic Molecules," Marcel Dekker, New York, N. Y., 1968, Chapter 2.

(33) P. A. Straub, D. Meuche, and E. Heilbronner, *Helv. Chim. Acta*, 49, 517 (1966).

(34) M. L. Heffernan and A. J. Jones, *Chem. Commun.*, 120 (1966).

Dewar benzene cannot be included in the comparisons, since there are no identifiable  $\pi$  orbitals.

The quantitative description also results in fulvene taking on an interesting role. In particular, its aromatic character is less than that of the three- and four-membered isomers, even though its total stability is greater than that of the other isomers. Consequently, the chemical characteristics of the  $\pi$  electrons in fulvene should exhibit more similarities to those of hexatriene than the other isomers.

A perhaps more convenient quantitative measure of aromaticity can be obtained if the open-chain analog is used as a reference molecule. In this case, one can define the aromaticity ( $A$ ) of a given molecule ( $M$ ) as

$$A = \left[ \frac{E_{\pi}(M) - E_{\pi}(\text{ref})}{E_{\pi}(\text{ref})} \right] 100 \quad (6)$$

where  $E_{\pi}(M)$  and  $E_{\pi}(\text{ref})$  are the sum of the occupied  $\pi$ -orbital energies for the molecule in question and the open-chain reference molecule, respectively. Using this measure of aromaticity, the benzene isomer aromaticities are summarized in Table X. Also given for

Table X. Aromaticities of Benzene Isomers

Molecule	Aromaticity ( $A$ ) <sup>a</sup>	Delocalization <sup>b</sup> Energy <sup>c</sup>
Benzene	11.073	36
Fulvene	1.032	26
2,3-Dimethylenecyclobutene	3.506	22
Trimethylenecyclopropane	2.444	23
Hexatriene	0.000	

<sup>a</sup> Calculated using eq 6. <sup>b</sup> Calculated using  $\beta = 18$  kcal/mol. <sup>c</sup> See ref 8.

comparison are delocalization energies, computed using Hückel theory.<sup>8</sup>

It might also be noted in passing that the terms "aromatic" and "nonaromatic" are not readily separable using this definition, except at the extremes. This is believed to be highly desirable, for it does not require an absolute decision that a particular molecule must be *either* aromatic *or* nonaromatic, but allows for an appropriate gradation of aromaticity that is more characteristic of the real molecules.<sup>35</sup>

The naphthalene isomers provide similar examples of the above notion of aromaticity, as revealed by examination of Tables V and VI. Qualitatively, three  $\sigma$  orbitals separate the lowest  $\pi$  orbital from the others in naphthalene, while in azulene two  $\sigma$  orbitals are present between the  $\pi$  orbitals and only one  $\sigma$  orbital is present

(35) For a definition of antiaromaticity similar in spirit to the proposed definition, see R. Breslow, *Chem. Brit.*, 4, 100 (1968), and references contained therein.

in fulvalene between the  $\pi$  orbitals. As noted earlier, the  $9a_g$ ,  $8b_{3u}$ , and  $1b_{1u}$  ( $\pi$ ) orbitals have very nearly degenerate energies, and more extensive basis set calculations may lower the relative energy of the  $1b_{1u}$  ( $\pi$ ) orbital so that its aromaticity is qualitatively more like benzene. However, the quantitative sums over orbital energies in Table V indicate the clear aromaticity decrease from naphthalene to fulvalene. Consequently, even though the orbital structure of fulvalene is quite similar to that of naphthalene, the particular environment in fulvalene does not produce a stabilization of the  $\pi$  orbitals as in the case of naphthalene and the resulting  $\pi$  electrons are expected to be considerably more reactive than those in naphthalene,<sup>36,37</sup> although in reactions similar to those found for naphthalene.

Further investigation of the similarities and differences of these isomers is underway, and various molecular properties (*e.g.*, dipole moments), molecular orbital contours, and a complete population analysis will be reported later.

## Conclusions

The studies on the benzene and naphthalene isomers and hexatriene using the molecular-fragment approach allow several conclusions of both theoretical and chemical interest.

(1) The method provides a reliable and computationally convenient means of electronic structure determination of large molecules.

(2) Examination of the filled  $\pi$  orbitals of the benzene and naphthalene isomers provides both a qualitative and quantitative rationalization of aromaticity. However, both  $\sigma$  and  $\pi$  orbitals must be considered explicitly to maintain the usefulness of the concept.

(3) The relative stability of benzene isomers is found to be benzene > fulvene > 2,3-dimethylenecyclobutene > trimethylenecyclopropane > Dewar benzene. Only benzene has been found to have considerable aromaticity.

(4) The relative stability of naphthalene isomers is found to be naphthalene > azulene > fulvalene. The molecular orbital structure of fulvalene is found to closely resemble that of naphthalene, but the molecule is considerably less aromatic.

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